

Cosmon inflation

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The cosmon field responsible for dynamical dark energy can induce an inflationary period in early cosmology. Details depend on dilatation symmetry breaking terms in its potential and kinetic term that are relevant only during early stages of the evolution of the universe. We discuss a model with almost static geometry in the Jordan frame, where the cosmic evolution of the Hubble parameter is replaced by a variation of the Planck mass and particle masses. This model predicts a spectral index $n = 0.97$ and a ratio for tensor to scalar fluctuations $r = 0.13$.

An inflationary period [1–6] in very early cosmology and the dominance of dark energy in very late cosmology are central elements of our present understanding of the universe. They are tested jointly by anisotropies in the cosmic microwave background (CMB) [7]. Both the very early and the very late stage of the cosmological evolution look actually very similar: the energy density of the universe is dominated by an almost static component with equation of state $p \approx -\rho$. Inflation is usually described by a scalar field, the inflaton. Here the necessity of a dynamical field arises since the inflationary period has to end. The proposal of dynamical dark energy [8] is also based on the evolution of a scalar field, the cosmon. An obvious question asks if the inflaton and the cosmon could be the same field χ . In principle, this is well possible, as argued in ref. [9].

Inflation and dark energy domination are related to very different epochs in cosmology and correspond to different values of the scalar field χ . The properties of the effective action for χ , i.e. potential and kinetic term, typically depend on the value of χ and can therefore be quite different for the two epochs. Based on this, one may tailor an action with suitable properties. It should account for (i) inflation, (ii) entropy production after inflation, (iii) a subdominant scalar energy density during the radiation and matter dominated epochs, (iv) domination of the energy density by the cosmon in the recent cosmological epoch. A good candidate for this sequence of epochs is a scaling solution [8], [10] for the radiation and matter dominated period. For such a scaling solution the ratio of scalar energy density ρ_h to the dominant radiation or matter component ρ_r or ρ_m remains constant. Thus ρ_h can be associated with “early dark energy” [11], with a fixed fraction $\Omega_h = \rho_h/\rho_c$ of perhaps a few percent. The long duration of the scaling, where ρ_h decreases with the inverse second power of time, can explain the small amount of the present value of ρ_h in units of the Planck mass, $\rho_h/M^4 \approx 10^{-120}$.

In this scenario we can divide the history of the universe in four periods. A first scalar field dominated period (inflation) is followed by the radiation and matter dominated periods for which the scalar field is subdominant and follows a scaling solution. Finally, a new scalar field dominated period has begun rather recently. All three transitions in this sequence need some reason for the end of the preceding epoch. For inflation it is typically the end of the

“slow roll” property. The end of radiation domination is triggered by matter becoming more important since it is diluted more slowly by the cosmic expansion. Finally, the end of the matter dominated period can be associated to an effective stop of the evolution of the scalar field. This results in constant ρ_h and ends the scaling solution, such that ρ_h/ρ_m increases. The stop of the scalar field may result from a qualitative change of the potential or kinetic term for a characteristic value of χ . Alternatively, it could be induced by a cosmological trigger event, as neutrinos becoming non-relativistic in growing neutrino quintessence [12, 13] or by the formation of non-linear structures.

For the description of dynamical dark energy by the cosmon the symmetry of dilatations or scale transformations plays an important role. The cosmon is the “pseudo-Goldstone boson” of spontaneously broken dilatation symmetry. Its potential arises from effects breaking the dilatation symmetry explicitly (“dilatation anomaly”). During the scaling solution these effects become less and less important. For infinite time a fixed point with exact dilatation symmetry is approached. At the fixed point, the cosmon would be an exactly massless Goldstone boson. We can now move backwards in cosmology. Then the scale symmetry breaking terms in the action become more and more important. They may therefore play a crucial role in very early cosmological epochs as inflation.

In this note we demonstrate that scale symmetry breaking terms in the scalar potential can indeed be responsible for an inflationary period. They are associated with a characteristic intrinsic mass scale μ . (Parameters in the action with dimension of mass or length break the scale symmetry explicitly.) A second important effect is a shift of the effective kinetic term for the cosmon at some characteristic scale \bar{m} . We investigate models where the coefficient of the kinetic term $\partial^\mu \chi \partial_\mu \chi$ is large for $\chi \ll \bar{m}$, and small for $\chi \gg \bar{m}$. The end of inflation will typically occur for $\chi \approx \bar{m}$. Details of the inflationary period depend, in general, on the detailed structure of the scale symmetry breaking terms.

We concentrate on a class of very simple models with a scalar potential $\tilde{V} = \mu^2 \chi^2$, where we use a Jordan frame with effective Planck mass depending on χ . For this model the curvature scalar \tilde{R} stays almost constant for the whole history of the universe, $\tilde{R} \approx \mu^2$, with $\mu = 2 \cdot 10^{-33} \text{eV}$. Only the effective Planck mass increases, being proportional to χ for the relevant cosmological epochs. Also parti-

cle masses scale proportional to χ , such that dimensionless ratios of masses and dimensionless couplings stay constant. All bounds from time variation of fundamental constants or apparent violations of the equivalence principle are obeyed. We find that this simple class of models is quite predictive for the slow roll parameters of inflation. It predicts for the spectral index $n = 0.97$ and for the ratio of tensor to scalar fluctuations $r = 0.13$.

The quantum effective action for the cosmon-graviton system is assumed to take the form

$$S_{cg} = \int_x \sqrt{g} \left\{ \frac{1}{2} K(\chi) \partial^\mu \chi \partial_\mu \chi + \tilde{V}(\chi) - \frac{1}{2} F(\chi) \tilde{R} \right\}, \quad (1)$$

with \tilde{R} the curvature scalar of the metric $\tilde{g}_{\mu\nu}$ and

$$\tilde{V}(\chi) = \mu^2 \chi^2 + \bar{\lambda}_c, \quad (2)$$

$$F(\chi) = \chi^2 + m^2. \quad (3)$$

For large χ^2 we can neglect m^2 in F such that χ plays the role of a variable (reduced) Planck mass [14]. This region will be reached for late cosmology. With the potential (2) and for suitable K this entails a quintessence cosmology [8], see below. In contrast, for small χ^2 the effective Planck mass and the potential become independent of χ . This region is assumed to dominate the very early stage of cosmology. For all χ we may associate $F^{-1}(\chi)$ with an effective gravitational constant. Higher order terms in an expansion of $m^2/F(\chi)$ or $\mu^2/F(\chi)$ could be added to $\tilde{V}(\chi)$. We omit them because they do not change the qualitative picture. For the cosmon kinetic term we choose

$$K(\chi) = 8\omega + 12\tau \frac{m^2}{F(\chi)}, \quad (4)$$

omitting again higher orders in m^2/F . For $\chi \rightarrow \infty$ one has $K(\chi) \rightarrow 8\omega$, with ω corresponding to a similar parameter in a Brans-Dicke theory [15]. Stability requires $\omega > -3/2$, with $\omega = -3/2$ singled out as the “conformal point”. We choose $\tau > -2\omega/3$ in order to guarantee stability also for finite χ and negative $\omega > -3/2$.

The field equations derived by variation of the effective action (1) show some unusual features due to the χ -dependence of the coefficient of the curvature scalar \tilde{R} . In particular, the cosmological value $\chi(t)$ can *increase* with time despite the fact that the minimum of the effective potential $\tilde{V}(\chi)$ occurs at $\chi = 0$ [8, 14].

The qualitative evolution is followed most easily if we perform a field transformation from the “Jordan frame” to the “Einstein frame”. For this purpose we employ a rescaling of the metric (for details see ref. [14])

$$\tilde{g}_{\mu\nu} = w^2 g_{\mu\nu}, \quad w^2 = M^2/F(\chi). \quad (5)$$

Expressed in the new “field coordinates” $g_{\mu\nu}$ the action reads

$$S_{cg} = \int \sqrt{g} \left\{ \frac{2M^2 k^2(\chi)}{\chi^2} \partial^\mu \chi \partial_\mu \chi + V(\chi) - \frac{M^2}{2} R \right\}, \quad (6)$$

with

$$V(\chi) = \frac{M^4 \tilde{V}(\chi)}{F^2(\chi)} = \frac{M^4 (\mu^2 \chi^2 + \bar{\lambda}_c)}{(\chi^2 + m^2)^2} \quad (7)$$

and

$$\begin{aligned} k^2(\chi) &= \frac{\chi^2}{8} \left\{ \frac{K}{F} + 3 \left(\frac{\partial \ln F}{\partial \chi} \right)^2 \right\} \\ &= \frac{\chi^2}{\chi^2 + m^2} \left\{ \omega + \frac{3}{2} + \frac{3}{2} (\tau - 1) \frac{m^2}{\chi^2 + m^2} \right\}. \end{aligned} \quad (8)$$

For large χ one has

$$\begin{aligned} \lim_{\chi \rightarrow \infty} V(\chi) &= \frac{M^4 \mu^2}{\chi^2}, \\ \lim_{\chi \rightarrow \infty} k^2(\chi) &= k_\infty^2 = \omega + \frac{3}{2} = \frac{1}{\alpha^2}. \end{aligned} \quad (9)$$

In the Einstein frame the potential vanishes for $\chi \rightarrow \infty$, corresponding to an “asymptotically vanishing cosmological constant” [16]. We choose M to be equal to the present value of the scalar field $\chi_0 = \chi(t_0) = \chi(a = 1)$ and associate it with the present reduced Planck mass, $M^2 = 8\pi G_N$, $M = 2.44 \cdot 10^{27} \text{eV}$. If we identify $V(\chi_0)$ with the present dark energy density we obtain

$$\frac{V(\chi_0)}{M^4} = \frac{\mu^2}{M^2} \approx 7 \cdot 10^{-121}, \quad \mu = 2 \cdot 10^{-33} \text{eV}. \quad (10)$$

This implies that μ is of the order of the present Hubble parameter H_0 . Setting the scales is actually arbitrary. The only thing that matters is the ratio $\mu^2/\chi^2(t_0)$ that can become very small if $\chi(t)$ increases sufficiently fast and for a sufficient time. This is actually the case in this type of model.

Defining ($\mu > 0$)

$$\varphi = 2M \ln(\chi/\mu) \quad (11)$$

the scalar part of the action becomes for large χ

$$S_c = \int_x \sqrt{g} \left\{ \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi + M^4 \exp \left\{ -\frac{\varphi}{M} \right\} \right\}, \quad (12)$$

with “kinetial” $k^2(\varphi)$ approaching the constant (9) for $\varphi \rightarrow \infty$. One recovers quintessence with an exponential potential, which shows for the dominance of matter ($n = 3$) or radiation ($n = 4$) the well known scaling solution [16] with a constant early dark energy fraction

$$\Omega_h = n k_\infty^2 = \frac{n}{\alpha^2}. \quad (13)$$

Observational constraints on early dark energy require $k_\infty \lesssim 0.1$ [17–21]. Realistic cosmology needs an end of this scaling solution around redshift $z \approx 5$, for example as a consequence of a cosmological trigger event as for growing neutrino quintessence [12, 13].

We are mainly interested here in the early inflationary period of cosmology for not too large χ . In this region of

the field space the cosmon is associated with the inflaton. In order to be close to the usual normalization we employ

$$\tilde{\sigma} = \frac{2M}{m}\chi, \quad (14)$$

such that for $\chi \approx m$ one has $\tilde{\sigma} \approx M$. With this normalization the scalar part of the action (6) reads

$$S_c = \int_x \sqrt{g} \left\{ \frac{1}{2} Z(\tilde{\sigma}) \partial^\mu \tilde{\sigma} \partial_\mu \tilde{\sigma} + V(\tilde{\sigma}) \right\}, \quad (15)$$

with

$$\begin{aligned} V &= \gamma M^4 \left(1 + \frac{x}{\beta} \right) (1+x)^{-2}, \\ Z &= \frac{k^2}{x} = \frac{\omega}{1+x} + \frac{3(\tau+x)}{2(1+x)^2}. \end{aligned} \quad (16)$$

Here we have introduced the dimensionless variables and parameters

$$\begin{aligned} x &= \frac{\tilde{\sigma}^2}{4M^2} = \frac{\chi^2}{m^2}, \\ \gamma &= \frac{\bar{\lambda}_c}{m^4}, \quad \beta = \frac{\gamma m^2}{\mu^2} = \frac{\bar{\lambda}_c}{m^2 \mu^2}. \end{aligned} \quad (17)$$

For small values of γ the potential can be substantially smaller than M^4 . The canonically normalized scalar field σ is related to $\tilde{\sigma}$ by

$$\frac{\partial \sigma}{\partial \tilde{\sigma}} = Z^{1/2}(\tilde{\sigma}). \quad (18)$$

We can now determine the slow roll parameters of inflation in the usual way. One finds

$$\begin{aligned} \epsilon &= \frac{M^2}{2} \left(\frac{\partial \ln V}{\partial \sigma} \right)^2 = \frac{x}{2Z} \left(\frac{\partial \ln V}{\partial x} \right)^2 \\ &= \frac{x}{2\omega(1+x) + 3(\tau+x)} \left(2 - \frac{1+x}{\beta+x} \right)^2. \end{aligned} \quad (19)$$

For $\beta < 1/2$ one has a maximum of V at

$$x_{\max} = 1 - 2\beta, \quad (20)$$

which corresponds to $\epsilon(x_{\max}) = 0$. In this case the initial value of x should be larger than x_{\max} . For the sake of simplicity we mainly concentrate on $\beta \geq 1/2$, keeping in mind that $\beta = 1/2$ is an interesting point in parameter space where the potential around $x = 0$ is particularly flat. The second slow roll parameter obtains as

$$\begin{aligned} \eta &= \frac{M^2}{V} \frac{\partial^2 V}{\partial \sigma^2} = 2\epsilon + M^2 \frac{\partial^2 \ln V}{\partial \sigma^2} \\ &= 2\epsilon + \left(\frac{\partial \ln V}{\partial x} \right)^{-1} \frac{\partial \epsilon}{\partial x} \\ &= 2\epsilon - \left(\frac{2}{1+x} - \frac{1}{\beta+x} \right)^{-1} \frac{\partial \epsilon}{\partial x}. \end{aligned} \quad (21)$$

It is typically of a similar size as ϵ .

For $\beta \geq 1/2$ the maximum of V is at $x = 0$. In the region around this maximum ϵ is always small,

$$\epsilon = \frac{4x}{2\omega + 3\tau} \left(\frac{\beta - \frac{1}{2}}{\beta} \right)^2, \quad (22)$$

while η obeys

$$\eta = -\frac{2}{2\omega + 3\tau} \frac{\beta - \frac{1}{2}}{\beta}. \quad (23)$$

Thus also η is small provided τ is large enough or β is close to $1/2$. For $\beta < 1/2$ a similar situation arises close to the maximum of V at x_{\max} . For simplicity we may take τ large enough such that the slow roll condition is always fulfilled for $x \lesssim 1$. There is no problem to obtain a sufficient duration of inflation. On the other hand, for very large x one has

$$\epsilon = \frac{1}{2\omega + 3} = \frac{1}{2k_\infty^2} \quad (24)$$

Since k_∞^2 must be small according to eq. (13) we conclude that the slow roll region has to end before the asymptotic behavior for $x \rightarrow \infty$ is reached. Typically, the end of the inflationary period may be situated around x_f with $\epsilon(x_f) \approx 1$.

In the slow roll approximation the amplitude of scalar density fluctuations generated during the inflationary phase obeys

$$\Delta^2 = \frac{1}{24\pi^2 \epsilon} \frac{V}{M^4}, \quad (25)$$

where V and ϵ have to be evaluated for the value of σ at the time when the characteristic scale leaves the horizon. This happens around 60 e -foldings before the end of inflation. A realistic size of the fluctuations requires $\Delta^2 = 2.95 \cdot 10^{-9} A$, $A \approx 0.7$. The ratio of tensor to scalar fluctuations is given by

$$r = 16\epsilon. \quad (26)$$

By observation it is bound to $r \lesssim 0.2$. Finally, the spectral index

$$n = 1 - 6\epsilon + 2\eta \quad (27)$$

is found by observation as $n \approx 0.96$.

The observed amplitude Δ^2 allows us to determine the value of the cosmon field \bar{x} at the time when a characteristic fluctuation left the horizon. It has to obey $V(\bar{x})/(M^4 \epsilon(\bar{x})) \approx 5 \cdot 10^{-7}$. This value should correspond to about 60 e -folding before the end of inflation, $N(\bar{x}) \approx 60$, where

$$N(\bar{x}) = \int_{\bar{x}}^{x_f} \frac{Z}{d \ln V / d \ln x} dx = \int_{\bar{x}}^{x_f} \sqrt{\frac{Z}{2x\epsilon}} dx. \quad (28)$$

For large τ the relevant last phase of inflation occurs for $x \gg 1$, where

$$Z = \frac{3\tau}{2x^2} + \frac{k_\infty^2}{x} \quad (29)$$

and

$$\epsilon = \frac{D_\beta}{2Zx} = \frac{D_\beta}{3\tau + 2k_\infty^2 x}. \quad (30)$$

The factor D_β depends on x/β ,

$$D_\beta = \left(2 - \frac{1+x}{\beta+x}\right)^2, \quad (31)$$

and varies between one for large x/β and four for small x/β . For a simple analytic discussion one may take for D_β some suitably averaged constant value between one and four. With the condition $\epsilon(x_f) = 1$ inflation ends for

$$x_f = \frac{3\tau}{D_\beta(x_f) - 2k_\infty^2}, \quad (32)$$

consistent with $x_f \gg 1$ for $\tau \gg 1$. Furthermore, with eqs. (28), (29), (31) the association between x and the number of e -foldings before the end of inflation $N(x)$ becomes

$$\begin{aligned} N(x) &= \frac{1}{\sqrt{D_\beta}} \int_x^{x_f} Z(x) dx \\ &= \frac{1}{\sqrt{D_\beta}} \left\{ \frac{3\tau}{2} \left(\frac{1}{x} - \frac{1}{x_f} \right) + k_\infty^2 \ln \frac{x_f}{x} \right\}. \end{aligned} \quad (33)$$

For $N(\bar{x}) = 60$ one finds

$$\bar{x} \approx \frac{\tau}{40\sqrt{D_\beta}} \approx \frac{\sqrt{D_\beta}}{120} x_f, \quad (34)$$

such that during this last phase of inflation x increases by about two orders of magnitude from \bar{x} to x_f . The validity of the approximation $\bar{x} \gg 1$ requires $\tau \gg 100$. (One may keep in mind, however, that inflation can also be realized for $\bar{x} \approx 1$ or $\bar{x} < 1$.)

We can now extract the relevant parameters for the density fluctuations. Inserting the value (34) one finds

$$\begin{aligned} \epsilon &= \frac{\sqrt{D_\beta}}{120}, \quad r = \frac{2\sqrt{D_\beta}}{15}, \\ \eta &= 2\epsilon - \frac{1}{120}, \quad n = 1 - \frac{1}{60}(\sqrt{D_\beta} + 1). \end{aligned} \quad (35)$$

The observed amplitude obtains for

$$\frac{V(\bar{x})}{M^4} \approx 4\sqrt{D_\beta} 10^{-9}. \quad (36)$$

We distinguished two cases: For $\bar{x} \gg \beta$ one has $D_\beta = 1$, (case A), while for $\bar{x} \ll \beta$ one finds $D_\beta = 4$ (case B). The observed value of Δ^2 requires then

$$(A): \quad \frac{\gamma}{\beta} = \frac{\mu^2}{m^2} \approx 10^{-10} \tau, \quad (37)$$

or

$$(B): \quad \gamma = \frac{\bar{\lambda}_c}{m^4} \approx \frac{1}{8} \cdot 10^{-12} \tau^2. \quad (38)$$

The transition between the two regimes is located around $\beta \approx \tau/80$, with (A) realized for $\beta \ll \tau/80$ and (B) for $\beta \gg \tau/80$. We observe that the fluctuation amplitude is determined by the relative size of two scale-violating parameters: The scale violation in the potential is determined by $\mu^2(A)$ or $\bar{\lambda}_c(B)$, while the scale violation in the scalar kinetic term involves the parameter τm^2 . Realistic amplitudes for density fluctuations are found for a comparatively small scale violation in the potential, $\mu^2/\tau m^2 \ll 1$ of $\bar{\lambda}_c/(\tau m^2)^2 \ll 1$. This corresponds to a rather generic situation and does not need fine-tuning of parameters.

For the case (A) a good approximation takes $\bar{\lambda}_c = 0$ and omits the term m^2 in $F(\chi)$. During the inflationary period the parameter k_∞ or ω only mildly influences the end of the inflationary period, while it only induces tiny corrections for the properties of density fluctuations. The latter depend only on one dimensionless ratio, $\mu^2/(\tau m^2)$, which fixes the amplitude. The slow roll parameters are independent of this ratio, such that this scenario is rather predictive,

$$(A): \quad n - 1 = -0.033, \quad r = 4(1 - n) = 0.13. \quad (39)$$

The situation is analogous for the scenario (B) which is also predictive

$$(B): \quad n - 1 = -0.05, \quad r = 0.27. \quad (40)$$

The transition between (A) and (B) interpolates between the values (39) and (40). We conclude that our model predicts rather substantial primordial gravitational waves, and present bounds may already disfavor the scenario (B).

In the following we concentrate on scenario (A) which is particularly simple. In order to fix ideas, we may consider the parameters $\tau = 10^4, \beta = 0.5, m = 10^3 \mu$ for which (A) is a very good approximation. It is instructive to discuss this model in the Jordan frame, eqs. (1)-(4). For both the inflationary phase and the subsequent scaling solution the potential (2) is given by a simple “mass term” $\tilde{V} = \mu^2 \chi^2$. The inflationary phase and the scaling solution are only distinguished by the effective kinetic term for the cosmon. For the inflationary phase one has $K = 12\tau m^2/\chi^2$, while soon after the end of inflation the kinetic term is dominated by $K = 8\omega$, cf. eqs. (4), (29). Most interestingly, the Hubble parameter \tilde{H} in the Jordan frame remains essentially constant for the whole cosmological evolution. Indeed, for a slow evolution of the cosmon the curvature scalar $\tilde{R} = 12\tilde{H}^2$ obeys

$$\tilde{R} = \frac{4\tilde{V}}{\chi^2} = 4\mu^2. \quad (41)$$

This constant value is only slightly modified in the presence of radiation or matter. Indeed, for the scaling solution one has now

$$\tilde{R} = \frac{8\tilde{V}}{\Omega_h(1-w_h)\chi^2} = \frac{8\mu^2}{\Omega_h(1-w_h)}, \quad (42)$$

with $\Omega_h = nk_\infty^2$ the fraction of (early) dark energy and w_h its equation of state. Also in the present dark energy

dominated epoch eq. (42) remains valid, now with $\Omega_{h,0} \approx 0.73$, $w_{h,0} \approx -1$. We arrive at a remarkable new picture of cosmology. It is not the geometry that changes, but only the strength of the gravitational interaction. The very early inflationary cosmology and the present universe differ only by the value of the effective Planck mass. The effective Planck mass χ has increased from a value when density fluctuations have left the horizon,

$$\bar{\chi}^2 = m^2 \bar{x} = \frac{\tau m^2}{40}, \quad (43)$$

to the present value $\chi_0^2 = M^2$.

The dimensionful parameter in the cosmon kinetic term K sets a characteristic scale of our model,

$$\bar{m} = \sqrt{12\tau m^2}. \quad (44)$$

A second characteristic scale corresponds to μ , as given by eq. (10). The parameter μ^{-1} is therefore set by the present horizon. One could choose units with $\mu = 1$ and only discuss ratios of mass scales. The other dimensionful parameter is somewhat larger than μ , cf. eq. (37)

$$\bar{m} = \sqrt{\frac{12\tau m^2}{\mu^2}} \mu = \sqrt{12} \cdot 10^5 \mu = 7 \cdot 10^{-28} eV. \quad (45)$$

Both fundamental scales are much smaller than all known energy scales in particle physics. This is due to the very large value of the present Planck mass $M = \chi(t_0)$ in units of μ ,

$$\frac{M}{\mu} \approx 10^{60}. \quad (46)$$

All present masses of elementary particles are proportional to M , albeit often with a small proportionality constant of 10^{-16} or smaller.

We finally discuss the coupling of the cosmon to the particles of the standard model. They are responsible for the production of the entropy and the heating of the universe at the end of the inflationary period. We concentrate on the couplings between the Higgs-doublet \tilde{h} and the cosmon that we parametrize by an effective Higgs potential (after inclusion of all quantum fluctuations) of the form

$$\tilde{V}_h = \frac{1}{2} \lambda_h (\tilde{h}^\dagger \tilde{h} - \epsilon_h \chi^2)^2. \quad (47)$$

The dimensionless couplings λ_h and ϵ_h depend on the ratios $\tilde{h}^\dagger \tilde{h}/\chi^2$ and χ^2/m^2 . Asymptotic scale invariance for $\chi^2/m^2 \rightarrow \infty$ requires that these couplings do no longer depend on explicit mass scales in this limit, and we define

$$\begin{aligned} \bar{\lambda}_h(\tilde{h}^\dagger \tilde{h}/\chi^2) &= \lambda_h(\tilde{h}^\dagger \tilde{h}/\chi^2, \chi^2/m^2 \rightarrow \infty), \\ \bar{\epsilon}_h(\tilde{h}^\dagger \tilde{h}/\chi^2) &= \epsilon_h(\tilde{h}^\dagger \tilde{h}/\chi^2, \chi^2/m^2 \rightarrow \infty). \end{aligned} \quad (48)$$

For fixed χ^2 and varying $\tilde{h}^\dagger \tilde{h}$ these couplings are still running in the standard way, due to the coupling of the Higgs boson to the fermions and gauge bosons of the standard

model. For late cosmology we can replace λ_h and ϵ_h by the scaling limit (48).

The running of $\bar{\epsilon}_h$ is governed by an anomalous dimension

$$\partial_t \bar{\epsilon}_h = A_h \bar{\epsilon}_h, \quad t = \frac{1}{2} \ln(\tilde{h}^\dagger \tilde{h}/\chi^2). \quad (49)$$

This particular form of the flow equation is due to an effective “low energy dilatation symmetry” associated to the second order character of the electroweak phase transition at $\bar{\epsilon}_h = 0$. The low energy dilatation symmetry protects small values of $\bar{\epsilon}_h$, rendering them “technically natural” [22, 23]. The electroweak gauge hierarchy requires a tiny value $\bar{\epsilon}_h \approx 10^{-32}$. This has to hold for $\tilde{h}^\dagger \tilde{h}/\chi^2 \approx 10^{-32}$. Due to the small value of A_h (proportional to squared gauge and Yukawa couplings) a tiny value is also required for larger values of $\tilde{h}^\dagger \tilde{h}/\chi^2$. The running of $\bar{\lambda}_h$ obeys the usual flow equations in the standard model. A fixed point for quantum gravity or some other mechanism may impose $\bar{\lambda}_h(\tilde{h}^\dagger \tilde{h}/\chi^2 = 1) \ll 1$, resulting in the prediction for the mass of the Higgs boson $M_H \approx 126$ GeV [24, 25].

In the scaling limit the (partial) minimum of the Higgs potential occurs for $\tilde{h}_0 \sim \chi$, as determined by

$$\tilde{h}_0^\dagger \tilde{h}_0 = \bar{\epsilon}_h(\tilde{h}_0^\dagger \tilde{h}_0/\chi^2) \chi^2. \quad (50)$$

Therefore the Fermi scale and associated particle masses depend on time, being always proportional to χ . In consequence, dimensionless ratios, as electron mass over Planck mass, are time independent despite the time dependence of both the Planck mass and the Fermi scale. We also assume a constant value of the gauge couplings at some sliding scale $M_U \sim \chi$, for example the scale of grand unification, $\bar{\alpha}_s(M_U) = \text{const.}$, and similar for Yukawa couplings. Then the severe observational bounds on the time dependence of fundamental couplings are met. (For possible small deviations from exact scaling and their observational consequences through a time variation of couplings and apparent violations of the equivalence principle see ref. [14, 26].)

For the inflationary period the scaling limit $\chi^2/m^2 \rightarrow \infty$ does not yet apply, despite the fact that for large τ the ratio $x = \chi^2/m^2$ is already substantially larger than one. Let us assume that the x -dependence of λ_h and ϵ_h is generated by

$$\begin{aligned} y \frac{\partial}{\partial y} \lambda_h &= -B_\lambda (\lambda_h - \bar{\lambda}_h), \\ y \frac{\partial}{\partial y} \epsilon_h &= -B_\epsilon (\epsilon_h - \bar{\epsilon}_h), \quad y = \frac{\chi^2 + \mu^2}{\mu^2}. \end{aligned} \quad (51)$$

(The positive “anomalous dimensions” B_λ and B_ϵ can depend on $\tilde{h}^\dagger \tilde{h}/\chi^2$.) For the approximation of constant B_λ and B_h the solution reads

$$\begin{aligned} \lambda_h &= \bar{\lambda}_h + \lambda_h^0 \left(\frac{\chi^2 + \mu^2}{\mu^2} \right)^{-B_\lambda}, \\ \epsilon_h &= \bar{\epsilon}_h + \epsilon_h^0 \left(\frac{\chi^2 + \mu^2}{\mu^2} \right)^{-B_\epsilon}. \end{aligned} \quad (52)$$

In particular, we observe that for $B_\epsilon \gtrsim 1/4$ one could have $\epsilon_h(x=0) = \epsilon_h^0 \approx 1$. The flow from $\chi^2 \approx \mu^2$ to $\chi^2 = M^2$

would realize in this case an early phase of attraction to an (approximate) fixed point at $\epsilon_h \approx 0$ in the spirit of ref. [25, 27]. (Note that ϵ_h should reach $\bar{\epsilon}_h$ before nucleosynthesis.) Indeed, we may assume that for $\chi^2 = \mu^2$ no particularly small or large parameters appear in the potential, with λ_h^0 and ϵ_h^0 of the order one.

The Weyl scaling (5) results in a multiplication of \tilde{V}_h by a factor w^4 , such that V_h obtains from \tilde{V}_h by replacing in eq. (47) λ_h by λ'_h ,

$$\lambda'_h = \frac{\lambda_h M^4}{(\chi^2 + m^2)^2}. \quad (53)$$

On the other hand, a standard kinetic term $\partial^\mu \tilde{h}^\dagger \partial_\mu \tilde{h}$ in the Jordan frame becomes in the Einstein frame

$$\begin{aligned} \mathcal{L}_h^{\text{kin}} &= \frac{M^2}{\chi^2 + m^2} \partial^\mu \tilde{h}^\dagger \partial_\mu \tilde{h} = \partial^\mu h^\dagger \partial_\mu h \\ &+ \frac{\chi^2 h^\dagger h}{(\chi^2 + m^2)^2} \partial^\mu \chi \partial_\mu \chi + \frac{\chi \partial^\mu \chi}{\chi^2 + m^2} \partial_\mu (h^\dagger h), \end{aligned} \quad (54)$$

with

$$h = \frac{M}{\sqrt{\chi^2 + m^2}} \tilde{h}. \quad (55)$$

Expressed in terms of h the effective potential for the Higgs doublet reads

$$V_h = \frac{1}{2} \lambda_h \left(h^\dagger h - \epsilon_h \frac{\tilde{\sigma}^2}{4 + \tilde{\sigma}^2/M^2} \right)^2. \quad (56)$$

During the inflationary phase we can neglect $\bar{\epsilon}_h$ and take $\chi^2 \gg \mu^2$. The partial minimum of the potential with respect to h obeys

$$h^\dagger h = \epsilon_h^0 \left(\frac{m^2 \tilde{\sigma}^2}{4\mu^2 M^2} + 1 \right)^{-B_\epsilon} \frac{\tilde{\sigma}^2}{4 + \tilde{\sigma}^2/M^2}, \quad (57)$$

defining a “valley” in the landscape of the potential. During the slow roll period of inflation the cosmological value of the Higgs doublet $h(t)$ follows this valley. A description in terms of coherent scalar fields remains valid. This changes after the end of inflation, when non-equilibrium processes result finally in the production of incoherent fluctuations of the Higgs fields, or equivalently, the production of an incoherent density of Higgs particles. In turn, the Higgs particles produce fermions and gauge bosons, producing in this way the entropy of the universe. The plasma of standard model particles is interacting sufficiently strongly such that thermal equilibrium can be established. At the end of this “heating of the universe” the cosmology is described by a radiation dominated Friedman universe, with an additional small part of early dark energy according to the scaling solution with $\Omega_h = nk_\infty^2$.

Details of entropy production and heating depend on the parameters of the model. One useful point of view is the

decay of cosmons into Higgs particles. The latter is directly induced by a cubic coupling $\gamma_h \chi h^\dagger h$ which obtains by taking a derivative of the effective action with respect to χ , h and h^\dagger . This cubic coupling depends on the cosmological background fields.

For small enough λ_h the influence of the Higgs doublet on the evolution of the cosmon is small during the inflationary period. For small ϵ_h also the field value of \tilde{h} is small compared to χ . For this reason we neglect a possible interaction $\sim \xi_h \tilde{h}^\dagger \tilde{h} \tilde{R}$ [8, 28]. (In a different setting, and for very large ξ_h , this term plays a crucial role in Higgs inflation [28] and Higgs-dilaton inflation [29–31].)

The coupling to the Higgs boson is only one of the possibilities for entropy production. The cosmon may couple as well to other scalar fields, as the ones responsible for spontaneous symmetry breaking of a grand unified symmetry. Furthermore, particle production could proceed during an epoch when the kinetic energy of the cosmon dominates [9, 32, 33],

In conclusion, we have found a rather simple setting for which the inflaton can be identified with the cosmon. While the cosmon-potential can be dominated by the same term $\tilde{V} = \mu^2 \chi^2$, a change in the scalar kinetic term from large K (inflation) to small K (scaling solution) can explain the end of inflation with a transition to the radiation dominated epoch. This class of models is predictive, leading to a spectral index $n = 0.97$ and an amplitude ratio for tensor to scalar fluctuations $r = 0.13$. One may ask if other relevant parameters for the behavior of early dark energy during the scaling solution can be connected to properties of inflation. This seems not to be the case in this class of models. The relevant parameter $k_\infty = 1/\alpha$ which fixes the scaling ratio of early dark energy $\Omega_{h,e}$ plays only a subdominant role during the inflationary stage. The parameters characterizing the present properties of dark energy, as the dark energy fraction $\Omega_{h,0} \approx 0.73$ or the equation of state $w_{h,0}$ will be mainly determined by the mechanism which stop the evolution of the cosmon at the end of the matter dominated period. Here the connection to the parameters which are relevant for the inflationary period seems even weaker.

The present dark energy density fixes $\mu = 2 \cdot 10^{-33} \text{eV}$ and therefore the value of the scalar potential for every value of χ . The fluctuation amplitude Δ depends, however, on the value χ_l when a fluctuation of a given length scale l has left the horizon. In turn, χ_l depends strongly on the parameter \bar{m} in the kinetic term. The kinetic term involves \bar{m} , giving an additional dependence of Δ on \bar{m} . On the other side, the terms involving \bar{m} are completely irrelevant for late cosmology and can therefore not be extracted from the present background cosmology. The spectral index and the scalar to tensor ratio are found to be independent of \bar{m} or k_∞ . This is the reason for the predictivity of these quantities for this class of models.

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- [1] A. H. Guth, Phys.Rev. **D23**, 347 (1981).
 - [2] A. Starobinski, Phys.Lett. **91B**, 99 (1980).
 - [3] A. D. Linde, Phys.Lett. **B108**, 389 (1982).
 - [4] A. Albrecht and P. J. Steinhardt, Phys.Rev.Lett. **48**, 1220 (1982).
 - [5] A. D. Linde, Phys.Lett. **B129**, 177 (1983).
 - [6] Q. Shafi and C. Wetterich, Phys.Lett. **B129**, 387 (1983).
 - [7] G. Hinshaw, D. Larson, E. Komatsu, D. Spergel, C. Bennett, et al. (2012), 1212.5226.
 - [8] C. Wetterich, Nucl.Phys. **B302**, 668 (1988).
 - [9] P. Peebles and A. Vilenkin, Phys.Rev. **D59**, 063505 (1999), astro-ph/9810509.
 - [10] B. Ratra and P. Peebles, Phys.Rev. **D37**, 3406 (1988).
 - [11] C. Wetterich, Phys.Lett. **B594**, 17 (2004), astro-ph/0403289.
 - [12] L. Amendola, M. Baldi, and C. Wetterich, Phys.Rev. **D78**, 023015 (2008), 0706.3064.
 - [13] C. Wetterich, Phys.Lett. **B655**, 201 (2007), 0706.4427.
 - [14] C. Wetterich, Nucl.Phys. **B302**, 645 (1988).
 - [15] C. Brans and R. Dicke, Phys.Rev. **124**, 925 (1961).
 - [16] C. Wetterich, Astron.Astrophys. **301**, 321 (1995), hep-th/9408025.
 - [17] M. Doran, G. Robbers, and C. Wetterich, Phys.Rev. **D75**, 023003 (2007), astro-ph/0609814.
 - [18] E. Calabrese, R. de Putter, D. Huterer, E. V. Linder, and A. Melchiorri, Phys.Rev. **D83**, 023011 (2011), 1010.5612.
 - [19] C. L. Reichardt, R. de Putter, O. Zahn, and Z. Hou, Astrophys.J. **749**, L9 (2012), 1110.5328.
 - [20] J. L. Sievers, R. A. Hlozek, M. R. Nolta, V. Acquaviva, G. E. Addison, et al. (2013), 1301.0824.
 - [21] V. Pettorino, L. Amendola, and C. Wetterich (2013), 1301.5279.
 - [22] C. Wetterich, Phys.Lett. **B140**, 215 (1984).
 - [23] C. Wetterich, DESY-87-154, C87/07/23 (1987).
 - [24] M. Shaposhnikov and C. Wetterich, Phys.Lett. **B683**, 196 (2010), 0912.0208.
 - [25] C. Wetterich, Phys.Lett. **B718**, 573 (2012), 1112.2910.
 - [26] C. Wetterich, JCAP **0310**, 002 (2003), hep-ph/0203266.
 - [27] C. Wetterich, Phys.Lett. **B104**, 269 (1981).
 - [28] F. Bezrukov, D. Gorbunov, and M. Shaposhnikov, JCAP **0906**, 029 (2009), 0812.3622.
 - [29] M. Shaposhnikov and D. Zenhausern, Phys.Lett. **B671**, 187 (2009), 0809.3395.
 - [30] J. Garcia-Bellido, J. Rubio, M. Shaposhnikov, and D. Zenhausern, Phys.Rev. **D84**, 123504 (2011), 1107.2163.
 - [31] F. Bezrukov, G. K. Karananas, J. Rubio, and M. Shaposhnikov (2012), 1212.4148.
 - [32] L. Ford, Phys.Rev. **D35**, 2955 (1987).
 - [33] B. Spokoiny, Phys.Lett. **B315**, 40 (1993), gr-qc/9306008.